Power Quality Indices for Electrical Power Systems under Non-Stationary Disturbances

P. A. Karafotis, K. N. Christodoulou-Galanopoulos, P. S. Georgilakis
National Technical University of Athens, School of Electrical and Computer Engineering
9, Iroon Polytechniou, Athens, Greece
panoskarafo@gmail.com

***Abstract*: This paper presents a methodology to evaluate power quality indices using Wavelet Packet Transform in electrical power systems, in the presence of harmonics, under stationary, non-stationary and short-circuit occurrence conditions, in order to achieve efficient monitoring of power systems. Results on several test systems and various disturbances simulated by Matlab/Simulink demonstrate the effectiveness and robustness of the proposed method.**

I. Introduction

The term power quality (PQ) indicates the deviation of both voltage and current from their ideal waveforms. An ideal waveform is considered to be sinusoidal, with fixed frequency and amplitude, any deviation from which is considered a disturbance. With the increased use of nonlinear loads, as well as due to time-varying single-phase and three-phase loads, the deformation of voltage and current from their ideal waveform has increased to a large extent. Therefore, the quantification of these deformations, using appropriate power quality indices (PQIs), has become a quite important issue in modern electrical power systems and offers the opportunity of efficient power systems monitoring. Total harmonic distortion of voltage and current, distortion index of voltage and current, as well as power factor, are worldwide established PQIs capable of compressing raw information, usually multidimensional in nature, into a single value [1].

Most PQIs currently used, are based on the analysis of voltage and current waveforms in their individual harmonic components. In order to analyze these disturbances and extract their spectrum, Fourier Transform (FT) is widely used [2]–[3]. However, using FT, signals can be described exclusively in the time or frequency domain, failing to provide information about time tracing of different frequencies within the signal. Thus, FT is no longer considered as the most suitable method for analyzing non-stationary disturbances containing time-varying frequency spectrum, such as short circuits. To address these problems, as well as to improve the existing PQIs and monitoring services, the wavelet transform (WT) is investigated. WT provides simultaneous time-frequency information of a signal, and as a result is more accurate in evaluating PQIs in electrical power systems with non-stationary disturbances [4]–[8].

In the present paper, Wavelet Packet Transform (WPT) is proposed to evaluate the PQ performance of single-phase and three-phase electrical power systems, operating under both stationary and non-stationary conditions. Typical PQ components, such as voltage, current and power, are reformulated by WPT and the most critical PQIs are calculated. Their values are presented and compared with their true values and those calculated with FT, in order to validate: 1) the ability of WPT to provide very accurate results in electrical power systems operating under both stationary and non-stationary conditions, 2) the advantage of WPT over FT to assess PQ performance in electrical power systems operating under non-stationary conditions, 3) the inability of FT to recognize short time disturbances.

This paper is organized as follows. Section II presents the WPT based mathematical approach to calculate PQIs. In Section III, the test cases and scenarios considered are described and the obtained results are presented. Conclusions are drawn in Section IV.

II. Mathematical Approach

A. Wavelet Packet Transform

Using a pair of filters, a low-pass $l[n]$ and a high-pass $h[n]$, the initial waveform given by the matrix $d_0^0[k]$, is analyzed into j decomposition levels. The initial matrix $d_0^0[k]$ contains 2^N samples and at each level j and node i, the WPT coefficients are derived from those of the previous level, by the convolution of the coefficients of the previous level with the low pass and the high pass filter, by the following equations, respectively:

$$d_j^{2i}[k] = \sum_n l[n] d_{j-1}^i [2k-n] \tag{1}$$

$$d_j^{2i+1}[k] = \sum_n h[n] d_{j-1}^i [2k-n] \tag{2}$$

where n is an internal variable of the convolution mathematical operation. Thus, in the final decomposition level, the signals $d_j^i[k]$ contain 2^{N-j} WPT coefficients and a frequency range of $\frac{f_{max}}{2}$, where f_{max} is the maximum frequency of the original signal. The maximum frequency is calculated using the sampling frequency f_s of the signal, based on the sampling theorem, by:

$$f_{max} = \frac{f_s}{2} \tag{3}$$

B. Voltage, Current and Power Formulation

The initial voltage and current disturbance is decomposed up to level j, in the way that each node includes exactly an odd harmonic order. Assuming that the WPT coefficients of each node i at the decomposition level j are $d_j^i[k]$ and $c_j^i[k]$ for voltage and current respectively, their RMS values V_j^i and I_j^i at each node i are computed by:

$$V_j^i = \sqrt{\frac{1}{2^N}\sum_{k=0}^{2^{N-j}-1}\left(d_j^i[k]\right)^2} \tag{4}$$

$$I_j^i = \sqrt{\frac{1}{2^N}\sum_{k=0}^{2^{N-j}-1}\left(c_j^i[k]\right)^2} \tag{5}$$

Using the RMS values of each node i, the total RMS values of voltage and current V_{rms} and I_{rms} are calculated by:

$$V_{rms} = \sqrt{\sum_{i=0}^{2^j-1}\left(V_j^i\right)^2} \tag{6}$$

$$I_{rms} = \sqrt{\sum_{i=0}^{2^j-1}\left(I_j^i\right)^2} \tag{7}$$

Active power P_j^i at each node i is given by:

$$P_j^i = \frac{1}{2^N}\sum_{k=0}^{2^{N-j}-1}\left(d_j^i[k]c_j^i[k]\right) \tag{8}$$

while the total active power P is computed by:

$$P = \sum_{i=0}^{2^j-1} P_j^i \tag{9}$$

The node zero apparent power S_j^0 is given by:

$$S_j^0 = V_j^0 I_j^0 \tag{10}$$

while the total apparent power S is calculated by:

$$S = V_{rms} I_{rms} \tag{11}$$

C. Power Quality Indices

Using the mathematical expressions of voltage, current and power of Section II.B, Total Harmonic Distortion of Voltage and Current, THD_V and THD_I, respectively, are given by:

$$THD_V = \frac{\sqrt{\sum_{i=1}^{2^j-1}\left(V_j^i\right)^2}}{V_j^0} = \sqrt{\frac{\sum_{i=1}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(d_j^i[k]\right)^2}{\sum_{k=0}^{2^{N-j}-1}\left(d_j^0[k]\right)^2}} \tag{12}$$

$$THD_I = \frac{\sqrt{\sum_{i=1}^{2^j-1}\left(I_j^i\right)^2}}{I_j^0} = \sqrt{\frac{\sum_{i=1}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(c_j^i[k]\right)^2}{\sum_{k=0}^{2^{N-j}-1}\left(c_j^0[k]\right)^2}} \tag{13}$$

Distortion Index DIN_V and DIN_I is defined as the ratio of the vector sum of the rms value of each node except node zero, to the total rms value of the original waveform, as:

$$DIN_V = \frac{\sqrt{\sum_{i=1}^{2^j-1}\left(V_j^i\right)^2}}{V_{rms}} = \sqrt{\frac{\sum_{i=1}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(d_j^i[k]\right)^2}{\sum_{i=0}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(d_j^i[k]\right)^2}} \tag{14}$$

$$DIN_I = \frac{\sqrt{\sum_{i=1}^{2^j-1}\left(I_j^i\right)^2}}{I_{rms}} = \sqrt{\frac{\sum_{i=1}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(c_j^i[k]\right)^2}{\sum_{i=0}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(c_j^i[k]\right)^2}} \tag{15}$$

Power factor PF can be calculated by:

$$PF = \frac{P}{S} = \frac{\frac{1}{2^N}\sum_{i=0}^{2^j-1}\sum_{k=0}^{2^{N-j}-1}\left(d_j^i[k]c_j^i[k]\right)}{\sqrt{\sum_{i=0}^{2^j-1}\left(V_j^i\right)^2}\sqrt{\sum_{i=0}^{2^j-1}\left(I_j^i\right)^2}} \tag{16}$$

while node zero power factor PF_{NZ} is defined as the ratio of the active power of node zero to the apparent power of node zero:

$$PF_{NZ} = \frac{P_j^0}{S_j^0} \tag{17}$$

III. Case Studies and Results

The proposed methodology has been tested in three different case studies, including representative scenarios, in order to examine several possible operating conditions of electrical power systems, as shown in Table I.

TABLE I
CASES STUDIES AND SCENARIOS CONSIDERED

Case study	Scenario	Input source	Input waveform
C1	S1	time domain equation	stationary
	S2	time domain equation	non-stationary
C2	S1	single-phase power system	stationary
	S2	single-phase power system	non-stationary
	S3	single-phase power system	non-stationary (short-circuit)
C3	S1	three-phase power system	stationary
	S2	three-phase power system	non-stationary
	S3	three-phase power system	non-stationary (short-circuit)

A. Case Study C1

In the first case study (C1), two scenarios (S1 and S2) are considered, in which the time domain equation of the waveforms are known. In S1, voltage and current waveforms are stationary, while in S2, non-stationary. In both S1 and S2, the fundamental frequency of the waveforms is 50 Hz, the observation period is 0.2 s and the waveforms are decomposed up to the 4th level, so that each frequency band corresponds to an odd harmonic component.

In S1, stationary voltage and current waveforms are simulated, containing 1st, 3rd, 5th, 7th, 9th, 11th, and 13th harmonic component, as follows:

$$\begin{aligned} v(t) = \sqrt{2}\cdot\{ & 230\sin(2\pi ft) + 46\sin(2\pi 3ft + 135°) + \\ & 46\sin(2\pi 5ft + 150°) + 23\sin(2\pi 7ft + 140°) + \\ & 18,4\sin(2\pi 9ft + 40°) + 23\sin(2\pi 11ft + 15°) + \\ & 23\sin(2\pi 13ft + 150°)\} \end{aligned} \tag{18}$$

$$\begin{aligned} i(t) = \sqrt{2}\cdot\{ & 10\sin(2\pi ft + 10°) + \sin(2\pi 3ft + 150°) + \\ & 0,8\sin(2\pi 5ft + 135°) + 0,8\sin(2\pi 7ft - 22,5°) + \\ & 0,9\sin(2\pi 9ft + 20°) + 0,7\sin(2\pi 11ft + 45°) + \\ & 0,8\sin(2\pi 13ft + 120°)\} \end{aligned} \tag{19}$$

In S2, the amplitude of the fundamental component of voltage waveform is $240\sqrt{2}$ for up to $t = 0.08$ s and then drops to $72\sqrt{2}$ until the end of the simulation (0.2 s). Apart from the fundamental component, voltage waveform also contains 5th and 7th harmonic components with amplitudes $60\sqrt{2}$ and $40\sqrt{2}$ respectively, throughout the observation period. The current waveform contains the fundamental component with an amplitude of $50\sqrt{2}$ up to $t = 0.08$ s, $15\sqrt{2}$ for $t \in [0.08, 0.13)$ and once again $50\sqrt{2}$ until the end of the simulation. It also contains 3rd, 5th and 7th harmonic components up to $t = 0.13$s, with amplitudes $12.5\sqrt{2}$, $10\sqrt{2}$ and $7.5\sqrt{2}$, respectively, and then they drop to zero.

The true values of PQIs for both S1 and S2 of C1, as well as the values calculated by WPT and FT are presented in Table II. The percentage difference between true values and the values calculated by WPT and FT are presented in Table III.

TABLE II
POWER QUALITY INDICES FOR CASE STUDY C1

PQIs	S1			S2		
	True values	**WPT**	**FT**	**True values**	**WPT**	**FT**
THD_V	0.3412	0.3429	0.3435	0.4461	0.4477	0.5237
THD_I	0.2054	0.2070	0.2079	0.3243	0.3262	0.4113
DIN_V	0.3229	0.3244	0.3248	0.4074	0.4086	0.4640
DIN_I	0.2012	0.2027	0.2035	0.3085	0.3101	0.3804
PF_{NZ}	0.9848	0.9848		0.8750	0.8792	
PF	0.9565	0.9565		0.8320	0.8320	

B. *Case Study C2*

In the second case study (C2), a single-phase circuit is considered, simulated by Matlab/Simulink, including three different loading scenarios (S1, S2 and S3). The circuit consists of a 24V DC voltage source, a DC/AC inverter controlled by SPWM control technique, and a transformer to increase the voltage to 230 V, as shown in Fig. 1.

TABLE III
PERCENTAGE DIFFERENCE OF POWER QUALITY INDICES FOR CASE STUDY C1

PQIs	S1		S2	
	WPT	**FT**	**WPT**	**FT**
THD_V	0.50 %	0.67 %	0.36 %	17.40 %
THD_I	0.78 %	1.20 %	0.59 %	26.83 %
DIN_V	0.46 %	0.59 %	0.29 %	13.80 %
DIN_I	0.75 %	1.14 %	0.52 %	23.31 %

In S1, a load of 9 kW is connected throughout the whole simulation period, thus voltage and current waveforms are stationary. In S2, two loads are connected to the circuit, a load of 8 kW, and a load of 1.5 kW and 1.5 kVar, for different amount of time, in order to examine how FT and WPT respond under non-stationary conditions. The first load is connected to the circuit during $t \in [0, 0.206] \cup [0.266, 0.4]$, while the second during $t \in [0.16, 0.4]$, In S3, the same loads of second scenario are connected to the circuit, however, a short-circuit is added for a brief amount of time during $t \in [0.320, 0.322]$. The values of PQIs calculated by WPT and FT for the three scenarios (S1, S2 and S3) of C2, as well as the percentage difference between WPT and FT, are presented in Table IV.

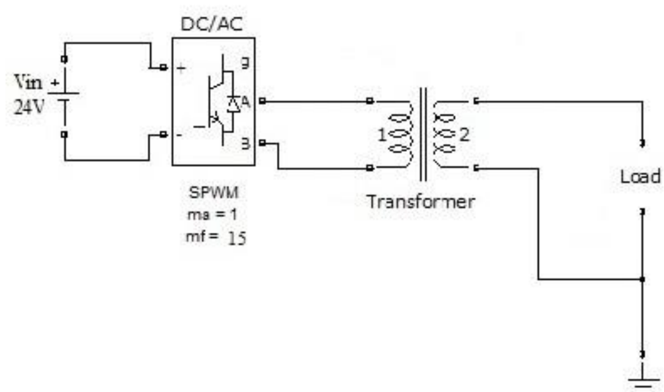

Figure 1. Single-phase power system for case study C2

TABLE IV
POWER QUALITY INDICES FOR CASE STUDY C2

Scenario	PQIs	WPT	FT	Difference %
S1	THD_V	0.3026	0.3026	0.00
	THD_I	0.3026	0.3026	0.00
S2	THD_V	0.3639	0.3321	9.58
	THD_I	0.2978	0.2825	5.42
S3	THD_V	0.3683	0.3316	11.08
	THD_I	0.3316	0.2838	16.84

C. Case Study C3

The third case study (C3) includes a symmetrical three-phase circuit, also simulated by Matlab/Simulink, examining three loading scenarios (S1, S2 and S3) similar to those in C2. As shown in Fig. 2, the circuit consists of a 48 V DC voltage source, a three-phase bridge inverter controlled by SPWM control technique, and a step-up transformer.

In S1, a symmetric three-phase load of 15 kW is connected to a grounded earth connection throughout the observation period. In S2, a three-phase load of 15 kW, during $t \in [0, 0.8] \cup [0.104, 0.24]$, and a symmetrical three-phase load of 10 kW and 6.2 kVar during $t \in [0.12, 0.24]$ are connected to the circuit. In S3, the same loads, as in S2, are connected to the circuit, but also a symmetrical three-phase short circuit during $t \in [0.18, 0.1824]$ is added.

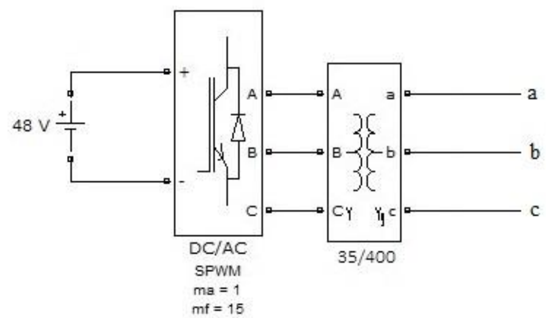

Figure 2. Three-phase power system for case study C3

The values of PQIs calculated by WPT and FT for the three scenarios (S1, S2 and S3) of C3, as well as the percentage difference between WPT and FT, are presented in Table V.

TABLE V
POWER QUALITY INDICES FOR CASE STUDY C3

Scenario	PQIs	WPT	FT	Difference %
S1	THD_V	0.2452	0.2440	0.49
	THD_I	0.2452	0.2440	0.49
S2	THD_V	0.3770	0.3304	14.10
	THD_I	0.2385	0.2376	0.38
S3	THD_V	0.4329	0.3397	27.44
	THD_I	0.3460	0.2425	42.68

IV. CONCLUSION

The proposed method for PQIs evaluation using WPT is tested on Matlab/Simulink simulated power systems and disturbances, both stationary and non-stationary, and the results obtained are compared with their true values and those by FT. The results validate that WPT is more suitable method than FT in evaluating power quality in electrical power systems operating under non-stationary conditions. Specifically, the following conclusions have been drawn from the above three case studies of Section III:

- In C1, it is verified that PQIs are accurately computed by the proposed WPT method, since their values are compared with their true ones, with the percentage difference being negligible and smaller than the difference between FT and true values.
- In S1, for both C2 and C3, where voltage and current waveforms are stationary, WPT and FT provide almost the same values for all PQIs.
- In S2, for both C2 and C3, where the loads are not constant, therefore voltage and current waveforms are non-stationary, WPT and FT provide quite different results, due to the disadvantage of FT to deal with time-varying loads.
- In S3, for both C2 and C3, with the addition of a transient phenomenon lasting for a very short period of time (short-circuit), the values of the PQIs computed by FT do not really change in comparison with S2, while by WPT are increased considerably. This result indicates that FT is not able to recognize short time highly non-stationary phenomena, like short-circuits, in contrast with WPT, where the values of PQIs changed significantly.

REFERENCES

[1] M. S. Kandil, S. A. Farghal, and A. Elmitwally, "Refined power quality indices," *IEEE Proc. Gener. Transm. Distrib.*, vol. 148, no. 6, pp. 590–596, Nov. 2001.

[2] *Electromagnetic compatibility (EMC) - Part 3-6: Limits - assessment of emission limits for the connection of distorting installations to MV, HV and EHV power systems*, IEC TR 61000-3-6, 2008.

[3] *Definitions for the measurement of electric quantities under sinusoidal, non-sinusoidal, balanced, or unbalanced conditions*, IEEE Standard 1459-2010, Mar. 2010.

[4] C. A. Naik and P. Kundu, "Power quality index based on discrete wavelet transform," *Int. J. Electr. Power Energy Syst.*, vol. 53, pp. 994–1002, Dec. 2013.

[5] K. Thirumala, A. C. Umarikar, and T. Jain, "Estimation of single-phase and three-phase power-quality indices using empirical wavelet transform," *IEEE Trans. Power Del.*, vol. 30, pp. 445–454, Feb. 2015.

[6] W. G. Morsi and M. E. El-Hawary, "Reformulating power components definitions contained in the IEEE Standard 1459-2000 using discrete wavelet transform," *IEEE Trans. Power Del.*, vol. 22, pp. 1910–1916, Jul. 2007.

[7] E. Y. Hamid, R. Maridiana, and Z. I. Kawasaki, "Method for RMS and power measurements based on the wavelet packet transform," *IEEE Proc. Sci. Meas. Technol.*, vol. 149, no. 2, pp. 60–66, Mar. 2002.

[8] W. G. Morsi and M. E. El-Hawary, "Novel power quality indices based on wavelet packet transform for non-stationary sinusoidal and non-sinusoidal disturbances," *Electr. Power Syst. Res.*, vol. 80, no. 7, pp. 753–759, Jul. 2010.